235131-NEP

251(N)

B. A./B. Sc. (Fifth Semester) EXAMINATION, 2024-25

MATHEMATICS

(Linear Algebra)

Time : Two hours] [Maximum Marks : 70

Notes: (i) *Attempt any* **five** *questions from Section (A) and any* **three** *questions from Section (B).*

(ii) *Answer each question of Section A within 50 words.*

(iii) *Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.*

Section-A

Note: *Attempt any* **five** *questions. Each question carries 5 marks.*

1. Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers. Define $(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$

$$c\,(x, y) \equiv (cx, y).$$

Is V, with these operations, a vector space over the field F?

(P.T.O.)

2. Are the vectors $\alpha_1 = (1, 1, 2, 4)$, $\alpha_2 = (2, -1, -5, 2)$, $\alpha_3 = (1, -1, -4, 0)$ and $\alpha_4 = (2, 1, 1, 6)$ linearly independent in IR^4?

3. Find the basis and dimensions of a vector space V (IF), where $V = \{A \mid A$ is a $m \times n$ matrix with complex entries$\}$ and IF = field of real numbers.

4. Show that the vectors $\alpha_1 = (1, 1, 0, 0)$, $\alpha_2 = (0, 0, 1, 1)$, $\alpha_3 = (1, 0, 0, 4)$, $\alpha_4 = (0, 0, 0, 2)$ form a basis for R^4. Find the co-ordinates of a vector $\alpha = (1, 0, 0, 0)$ in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

5. Which of the following function T from IR^2 into IR^2 are linear transformations?

(a) $T(x_1, x_2) = (1 + x_1, x_2)$;

(b) $T(x_1, x_2) = (x_1, 1 + x_2)$;

(c) $T(x_1, x_2) = (x_1, 0)$;

(d) $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$;

(e) $T(x_1, x_2) = (x_1, \cos x_2)$.

6. Let V and W be vector spaces over the field IF and let T be a linear transformation from V into W. Then, define the terms.

(a) Null space of T (b) Rage of T

(c) Nullity of T (d) Rank of T.

7. Let V be the vector space of all $n \times n$ matrices over the field IF, and let B be a fixed $n \times n$ matrix. If $T(A) = AB - BA$, then verify that T is a linear transformation from V into V.

Section-B

Note: *Attempt any* **three** *questions. Each question carries 15 marks.*

8\. (a) Let V and W be a vector space over the field IF. Let T be a linear transformation from V into W. Show that, for any $C \in$ IF, the function cT defined by

$$(cT)(\alpha) = c(T\alpha)$$

is a linear transformation from V into W.

(b) Let U, V and W be vector space over the field IF. Let T be a linear transformation from U into V and S a linear transformation from V into W. Then, show that the compared function ST defined by $ST(\alpha) = S(T\alpha)$ is a linear transformation from U into W.

9\. (a) Let V be the real vector space spanned by the rows of the matrix

$$A = \begin{bmatrix} 3 & 21 & 0 & 9 & 0 \\ 1 & 7 & -1 & -2 & -1 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 12 & -1 & 13 & 0 \end{bmatrix}.$$

Find a basis for V.

(b) Let V be a vector space over the field IF. The intersection of any collection of subspace of V is a subspace of V.

10\. If W_1 and W_2 are finite dimensional subspace of a vector space V, then prove that $W_1 + W_2$ is also a finite dimensional subspace of V, and

$$\dim W_1 + \dim W_2 = \dim(W_1 + W_2) + \dim(W_1 \cap W_2).$$

11. Which of the following sets of vectors $\alpha = (a_1, a_2, \ldots, a_n)$ in $\mathbb{R}^n$ are subspace of $\mathbb{R}^n$ $(n \geq 3)$?

(a) all α such that $a_1 \geq 0$;
(b) all α such that $a_1 + 3a_2 = a_3$;
(c) all α such that $a_1 + a_2 + \ldots + a_n = 0$;
(d) all α such that $a_n = a_1 + a_2 + \ldots + a_{n-1}$;
(e) all α such that $a_1 a_2 \ldots a_n = 0$.

12. (a) Find the matrix of a linear transformation T from IR^3 into IR^2, defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 + x_3)$ in the standard ordered basis for IR^3 and IR^2.

(b) Obtain a linear transformation T from IR^2 into IR^3 whose matrix in the standard ordered basis is given by $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -1 & -2 \end{bmatrix}_{3 \times 2}$.

13. (a) Find the eigen values and eigen vectors for the matrix $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$.

(b) Show that the linear operator T on IR^3 is diagonalizable for which the standard ordered basis matrix is given by

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}.$$