236131

253(N)

B.A./B.Sc. (Sixth Semester) Examination, 2024-25

MATHEMATICS

(Complex Analysis)

ime : Two Hours] [Maximum Marks

Iote : (i) Attempt any *five* questions from Section A an *three* questions from Section B.

(ii) Answer each question of Section A within 50 v

(iii) Limit your answers within the given answer Additional answer book (B-Answer book) sl not be provided or used.

(Section–A)

Note : Attempt any *five* questions. Each question car marks.

1. Evaluate the following limits :

(a) $\lim\limits_{z \to i} \sinh z$.

(b) $\lim_{z \to \infty} \frac{\sin z}{z}$.

. Express $\cosh(x + iy)$ and $\cos(x + iy)$ into the form $u(x, y) + iv(x, y)$.

. Which of the following satisfy the Cauchy-Reimann equations :

(a) $f(x, y) = (y^3 - 3x^2y) + i(x^3 - 3xy^2 + 2)$.

(b) $f(x + iy) = e^{\sin(x + iy)}$.

. If $f(x, y) = u(x, y) + iv(x, y)$ is an analytic function, where $u(x, y) = \cos x \cosh y$, then find the harmonic conjugate $v(x, y)$.

. Evaluate $\oint_C z\bar{z}\, dz$, where C is the unit circle enclosing the origin.

Find the Taylor series expansion around $z = 0$ of the function $f(z) = \frac{\sin z}{z}$ in the region $0 < |z| < \infty$.

Evaluate $\oint_C \frac{e^z}{z} dz$, where C is the unit circle centered at the origin.

(Section–B)

Note : Attempt any *three* questions. Each question carries 15 marks.

1. Determine whether the following functions are analytic. Discuss whether they have any singular points :

(a) $\tan z$.

(b) $e^{\sin z}$.

(c) $e^{1/(z-1)}$.

(d) $e^{\bar{z}}$.

(e) $\dfrac{z}{z^4+1}$.

2. (a) If a function $f(z)$ is analytic is a simply connected domain D, the prove that along a simple closed contour C in D :

$$\oint_C f(z)\,dz = 0.$$

(b) Let $f(z)$ be analytic interior to and one a simple closed contour C. If C is a circle, $|z-a| = \mathrm{R}$, and $|f(z)| \le \mathrm{M}$ $\forall\, z \in \mathrm{C}$, then prove that

$$\left|f^{(n)}(a)\right| \le \frac{n!\mathrm{M}}{\mathrm{R}^n}.$$

3. State and prove the Liouville's Theorem.

4. State and prove the Taylor series expansion of a comple: function $f(z)$.

5. Find the Laurent expansions for the following function i the given region :

(a) $f(z)=\dfrac{1}{1+z}$ for $|z|>1$.

(b) $f(z)=\dfrac{1}{(z-1)(z-2)}$ for $1<|z|<2$.

(c) $f(z)=\dfrac{1}{1+z^2}$ for $|z|>1$.