235131-NEP
251(N)

B.A./B.Sc. (Fifth Semester)
EXAMINATION, 2025-26

MATHEMATICS
(Linear Algebra)

me : Two Hours] [Maximum Marks : 70

te :

Attempt any five questions from Section (A) and three questions from Section B.

) Answer each question of Section (A) within 50 words.

i) Limit your answer within the given answer book Additional answer book (B-Answer book) should not b provided or used.

SECTION-A
(Short Answer Type Questions)

ote : Attempt any five questions. Each question carrie 5 marks.

Define Vector subspace. Prove or disprove that th set of all $n \times n$ matrices with their elements as rationa

number is a subspace over the field of real numbers.

2. Define linear and direct sum of two subspace.
If $W_1 = \{x, o, z\} : x,z \in IR\}$ and
$W_2 = \{(o, y, z) : y, z \in IR\}$ are
subspaces of $IR^3(IR)$ then prove or
disprove that $W_1+W_2 = R^3$ and $W_1 \oplus W_2 = IR^3$.

3. Prove that linear span of any non-empty subset of a vector space is a subspace. Also prove that $L(S) = IR^3$ where $S = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$.

4. Prove that every subset of linearly independent set is linearly independent. For which value of x, vector $(0, 1, x), (x, 1, 0), (1, x, 1)$ is linearly dependent.

5. If $T : U(F) \rightarrow V(F)$ be a linear transformation then range R(T) of T is a subspace of V(F). Find R(T) of $T : IR^2 \rightarrow R^2$ s.t. $T(x, y) = (x+y, x+y)$.

6. If W is a subspace of $IR^4(IR)$, generated by the vector $(3, 8, -3, -5), (1, -2, 5, -3), (2, 3, 1, -4)$, then find basis and dimension of W. Extend this basis to become a basis of $IR^4(IR)$

7. Defien dual space and dual basis. If $V^*(F)$ be dual space of a finite dimensional vector space V(F) then prove that dim V = dim V*.

SECTION-B

(Long Answer Type Questions)

Note : Attempt any three questions of the following. Each question carries 15 marks.

8. (a) Prove that the union of two subspaces of a vector space is a subspace iff one is contained in the other.

   (b) If W_1 and W_2 are two subspaces of a finite dimensional vector space V(F) then
   $\dim W_1 + \dim W_2 = \dim (W_1 + W_2) + \dim (W_1 \cap W_2)$

9. If W_1 and W_2 are subspaces of the vector space $IR^4(IR)$ generated by the sets $\{(1, 1, 0, -1), (1, 2, 3, 0), (2, 3, 3, -1)\}$ and $\{(1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)\}$ respectively then find $\dim (W_1 + W_2)$, $\dim (W_1 \cap W_2)$ $\dim (V/W_1)$, and $\dim (V/(W_1 \cap W_2))$.

10. If U(F) is a finite dimensional vector space and $T : U(F) \to V(F)$ is a linear transformation then
    rank (T) + nullify(T) = dim U
    Also find rank(T) is
    T is non-singular and dim(U) = 3

11. (a) Let $T : U(F) \to V(F)$ is a linear transformation. Then T is non singular iff T carries each linearly independent sub set of U onto a linearly independent subset of V.

(b) Let $T : IR^3 \to IR^3$ be a linear operator suc that $T(x, y, z) = (3x, x-y, 2x+y+z)$. Is invertible? If so, find a rule for T^{-1}.

12. If $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$ is matrix of

transformation (linear)

$T : IR^3(IR) \to IR^3(IR)$ with respect to basis $\{(1, 0, 0), (0, 1, 0), (0, 0, 1\}$ then, find linea transformation T and matrix of transformation wit respect to basis $\{(0, 1, -1), (1, -1, 1), (-1, 1, 0)\}$.

13. Find eigen value and vectors for a linear transform given by

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}.$$ Is a diagonalizable?

If so, find its diagonal form.