Roll No. ....

232311

S-248(A)-N

B.A./B.Sc. (Second Semester) (Fourth Semester)
NEP EXAMINATION 2023-24
MATHEMATICS
(Vector Calculus)
[SOS/Maths/SEC-I]

Time : Two Hours] *[Maximum Marks : 70*

Note: (i) Attempt any five questions from Section A and any three questions from Section B.

(ii) Answer each question of Section A within 50 words.

(iii) Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

Section-A

Attempt any five questions. Each question carries five marks.

1. Find the value of:

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}).$$

2. Prove that: $[a' \; b' \; c'] \, [\vec{a} \; \vec{b} \; \vec{c}] = 1$

   If $\vec{a}, \vec{b}, \vec{c}$ and a^{-1}, b^{-1}, c^{-1} are reciprocal system of vectors.

3. Prove that the necessary and sufficient condition for the vector $\vec{a}(t)$ to have constant magnitude is: $a. = \frac{d\vec{r}}{dt}$ 0

4. If $\phi = \log |\vec{r}|$, then show that:

$$\text{grad } \phi = \frac{\vec{r}}{r^2}, \text{ where } \vec{r} = x\hat{i} + y\hat{j} + 2\hat{k}$$

5. Show that the vector $\vec{F} = xyz^2 G$, Where $G = (2x^2+8xy^2z)\hat{i}+(3x^3y-3xy)\,j-(4y^2+z^2+2x^3z)\hat{k}$ is solenoidal.

6. If $\vec{\phi} = x^3 + y^2 + z^3 - 3xyz$, find the value of div(grad$\vec{\phi}$) and curl (grad$\vec{\phi}$)

7. If $\vec{a} = t\hat{i} - 3\hat{j} + 2t\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{c} = 3\hat{i} + t\hat{j} - \hat{k}$, show that:

$$\vec{a}.(\vec{b} \times \vec{c})dt = -14.$$

Section-B

8. (a) Show that the points whose position vectors are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ will be coplanar if;

$$[\vec{a}\,\vec{b}\,\vec{c}] - [\vec{a}\,\vec{b}\,\vec{d}] + [\vec{a}\,\vec{c}\,\vec{d}] - [\vec{b}\,\vec{c}\,\vec{d}\,] = 0$$

(b) Prove that if a, b, c and a, b, c are reciprocal system of vectors, then:

$$\vec{a} \times \vec{a} + \vec{b} \times \vec{b} + \vec{c} \times \vec{c} = 0$$

9. Find the first and second derivatives of the products:

(a) $\left[\vec{r}\ \dfrac{d\vec{r}}{dt}\ \dfrac{d^2\vec{r}}{dt^2}\right]$

(b) $\vec{r} \times \left[\dfrac{d\vec{r}}{dt} \times \dfrac{d^2\vec{r}}{dt^2}\right]$

where $\vec{r} = r\hat{i} + y\hat{j} + z\hat{k}$

10 (a) Prove that:

$$\text{curl}(u\vec{a}) = (\text{grad } u) \times \vec{a} + u \text{ curl } \vec{a}$$

(b) Prove that:

$$\text{div}(\vec{a} \times \vec{b}) = \vec{b} \text{ curl } \vec{a} - \vec{a} \text{ curl } \vec{b}$$

11. Show that necessary and sufficient condition that direction of given vector $\vec{r}$ is constant is that:

$\vec{r} \times d\vec{r}/dt = 0$, where $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$.

12. Verify divergence theorem for $\vec{F} = 4 \times z\hat{i} - y\hat{j} + yz\hat{k}$ taken over the cube bounded by x=0, x=1, y=0, y=1,z=0, z=1.

13. Verify stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and z is its boundary.

$(2\cos\theta - \sin\theta)$