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M. A./M. Sc. (First Semester) EXAMINATION, 2024-25

MATHEMATICS

Paper First

(Discrete Structures)

(MATH—001)

Time : Two hours] **[Maximum Marks : 60**

Note: *Attempt any **four** questions. All questions carry equal marks.*

1. Define Conjunctive Normal Form.

   Find the product of sum expansion (CNF) of each of the following. Boolean functions.

   (i) $f(x, y, z) = (x + y + z)\, z'$

   (ii) $f(x, y, z) = xyz$

   (iii) $f(x, y, z) = xy'z$

2. Write short notes on any four of the following with examples.

   (i) Eulerian Graphs

(P.T.O.)

(ii) Bipartite Graphs

(iii) Ring sum of the graphs

(iv) Complete graph

(v) Partially ordered sets

(vi) Lattices.

3. Define Recurrence Relations.

Solve the following Recurrence Relations.

(i) $a_n = 5a_{n-1} - 6a_{n-2}$ with the initial conditions $a_0 = 1, a_1 = 2$

(ii) $a_n = a_{n-1} + 2a_{n-2}; n \geq 2; a_0 = 0, a_1 = 1$

4. (a) Explain Karnaugh's map for three variables to use it to simplify the Boolean functions. Applying Karnaugh Map Simplify the following Boolean Expression.

$f(x, y, z) = x'y'z + x'yz + xyz + xy'z' + x'yz'$

(b) Discuss Graphs. Prove that the sum of the degree of vertices in an undirected graph is even. Give an example.

5. (a) State and prove Handshaking theorem of Graph theory and give some examples.

(b) Define Sub Boolean Algebra. Prove that for any a, b, c in a Boolean Algebra the followings are equal:

(i) $(a + b)(a' + c)(b + c)$,

(ii) $ac + a'b + bc$

(iii) $ac + a'b$

6. (a) Define Isomorphic Boolean Algebra. Let $B = \{0, 1\}$ and $B' = \{a, b\}$ be the two Boolean Algebra under the operations +, * and ' are defined by the following tables.

For $B = \{0, 1\}$

+	0	1
0	0	1
1	1	1

*	0	1
0	0	0
1	0	1

'	0	1
	1	0

and For $B' = \{a, b\}$

+	a	b
a	a	b
b	b	b

*	a	b
a	a	a
b	a	b

'	a	b
	b	a

Show that B is isomorphic to B'.

(b) Define complemented Lattice.

Show that two Bounded lattices L_1 and L_2 are complemented if and only if $L_1 \times L_2$ is complemented.

7. Define Generating functions for the sequence, Applying the Method of generating function solve the following recurrence relations

(i) $a_r - 7a_{r-1} + 10a_{r-2} = 0$

With the conditions $a_0 = 3$ and $a_1 = 3$

(ii) $a_{r+2} - 2a_{r+1} + a_r = 2^r$

With the initial conditions $a_0 = 2$ and $a_1 = 1$

8. Attempt any three of the following:

(i) If $A = \{a, b, c\}$, prove that Lattice $(P(A), \cap U)$ (under $\subseteq$) is distributive, Where $P(A)$ = Power set of A.

(ii) Define 'Hasse Diagram'. Draw the Hasse Diagram of the Relation R on A where $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)\}$

(iii) A graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in G.

(iv) If B be a Boolean Algebra for every element in a Boolean Algebra, Prove the following:

(a) $a + a = a$ and $a * a = a$

(b) $a + (a * b) = a$ and $a * (a + b) = a$.

(v) Define Self Complementry Graph.

If G is a self complementy Graph then prove that G has $4k$ or $4k + 1$ vertices, where k be an integer.